

A mass-deformed BLG theory in Gribov-Zwanziger framework

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In this paper, we extend the Gribov-Zwanziger framework accounting for the existence of Gribov copies to the mass deformed Bagger–Lambert–Gustavsson (BLG) theory in $\mathcal{N} = 1$ superspace. The restriction of the domain of integration in the Euclidean functional integral to the first Gribov horizon is implemented, by adding a non-local horizon term to the effective action. Furthermore, the soft breaking of the BRST symmetry due to horizon term is restored with the help of external sources. We compute the various Ward identities for this theory relying on the Lie 3-algebras.

Keywords: BLG theory; Gribov problem; BRST symmetry.

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I. INTRODUCTION

The understanding of M-theory is important from the view point that it can be the most profound unified theory for particle physics. So, it is essential to study the behavior of M-branes, a basic ingredient of M-theory. Dirac's prescription on monopoles suggests that charge of M-brane is quantized, i.e., the number of charges are countable. While the dynamics of a single M-brane is well understood, at least at classical level, a very little is known about the multiple M-branes. In such circumstances, a lot of excitations have been seen for a model of multiple M2-branes based on Lie 3-algebra proposed by Bagger, Lambert [1–3] and Gustavsson [4]. The Bagger–Lambert–Gustavsson (BLG) model describes a three dimensional superconformal field theory with $\mathcal{N} = 8$ supersymmetry, proposed as the world-volume action for two interacting M2-branes. In this model, the field content is a collection of scalars, fermions and gauge fields transforming under a Lie 3-algebra (a generalization of a Lie algebra with a triple bracket replacing the commutator and a 4-index structure constant replacing the usual 3-index structure constant of a Lie algebra).

Although the original BLG action possess high amount of supersymmetry [5–8], the prominence of simple (or $\mathcal{N} = 1$) superfields in three dimensions can never be underrated [9]. The superfield description of three dimensional BLG theory in $\mathcal{N} = 1$ superspace is described in Ref. [10]. The dimensional reduction of the multiple M2-branes in $\mathcal{N} = 1$ superspace formalism has been analysed [11]. In this context, a map to a Green-Schwarz string wrapping a nontrivial circle in $\mathbb{C}^4/\mathbb{Z}_k$ has also been constructed. Moreover, it has been observed that the mass deformation breaks the conformal invariance of the BLG theory though it preserves maximal supersymmetry [12, 13]. While this deformed theory has discrete vacua, its non-relativistic limit in the symmetric phase turns out to acquire a superconformal symmetry, different from the original mass-deformed ABJM model. For a given number of particles and antiparticles in mass-deformed scenario, we can consider the low energy physics where the speed of particles are much slower than that of the speed of light. As a consequence of the conformal breaking, there will exist a renormalization group invariant scale, it turns out that it is possible to attach a dynamical meaning to the Gribov parameter. This implies that, in mass deformed BLG theory, the restriction of the domain of integration in the functional integral to the Gribov region is important, and non-perturbative dynamical scale can be generated. Recently, the BLG theory in $\mathcal{N} = 1$ superspace has been quantized in Lorentz gauge [14–22]. Here, we study the BRST quantization of mass deformed BLG theory in $\mathcal{N} = 1$ superspace

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in Landau gauge.

According to standard quantization methods of a gauge theories, for almost all calculations aside from lattice simulations of gauge-invariant quantities, one needs to fix a gauge [23]. In certain choices, even after fixing the gauge, the redundancies of gauge fields do not disappear completely for large value of gauge fields (Gribov problem) [24]. The non-Abelian gauge theories in those gauges contain so-called Gribov copies, which play a crucial role in the infrared (IR) regime while it can be neglected in the perturbative ultraviolet regime [24–26]. Such investigations have become very exciting currently due to the fact that color confinement is closely related to the asymptotic behavior of the ghost and gluon propagators in deep IR regime [27]. In order to make the theory free from those copies, Zwanziger proposed a theory, commonly known as Gribov and Zwanziger (GZ) theory, by restricting the domain of integration in the functional integral with a (non-local) horizon term [25]. A composite fields approach has also been presented to eliminate Gribov copies from non-Abelian theories [28].

On the other hand, the horizon term of the GZ action breaks the BRST symmetry of the theory softly [29]. However, the Kugo-Ojima (KO) criterion for color confinement [30] is based on the assumption of BRST exactness of the theory. It has been shown that a consistent quantization of gauge theories with a soft breaking of BRST symmetry does not exist [31] and leads to inconsistency in the conventional BV formalism [32, 33]. Further, It has been established that the gauge theories with soft breaking of BRST symmetry can be made consistent if the transformed BRST-breaking terms satisfy certain requirement [34]. The BRST symmetry in presence of the Gribov horizon has great applicability in order to solve the non-perturbative features of confining Yang-Mills (YM) theories [35, 36], where the soft breaking of the BRST symmetry exhibited by the GZ action can be converted into an exact invariance [37, 38]. Also, the spontaneously broken Slavnov-Taylor identity has been converted to the linearly broken Ward identity under certain algebraic circumstances [39]. Subsequently, a nilpotent BRST transformation which leaves the GZ action invariant has been obtained and can be applied to KO analysis of the GZ theory [40]. Finite BRST-antiBRST transformations were also developed in the GZ context [38, 41]. A full resolution of the Soft breaking of BRST symmetry was done in Ref. [42]. The GZ treatment in R_ξ gauges was done for the standard model also. Recently, a gauge-invariant formulation of the GZ model for YM theory with local BRST transformations was given for the first time in Ref. [43]. Such investigations are very useful in order to evaluate the vacuum expectation value of BRST exact quantity.

In this work, we consider the mass deformed BLG theory $\mathcal{N} = 1$ superspace in Landau gauge. We derive the BRST symmetry for the theory. Furthermore, to discuss the non-perturbative regime, we implement the GZ framework to the theory by adding a non-local horizon term to the effective action which restricts the domain of integration of functional integral to the first Gribov horizon. We also localize the horizon term by extending the configuration space with quartet of auxiliary fields. Within formulation, we notice that the presence of γ -dependent terms break the BRST invariance of the BLG action. To restore the BRST invariance of the BLG theory in GZ framework, we introduce three more pairs of external sources with certain physical values. This symmetry turns out to be useful in order to establish non-perturbative Ward identities, allowing us to evaluate the vacuum expectation value of quantities which are BRST exact. Further, we compute the Ward identities corresponding to BRST exact action together with external sources. The present investigations will be helpful to compute the counter terms for the multiplicative renormalizability of the theory.

The paper is organized as follows. In Sec. II, we analyse the mass deformed BLG Theory in $\mathcal{N} = 1$ superspace in Landau gauge. This theory follows Lie 3-algebras. In Sec. III, we discuss the theory in GZ framework. Within this framework the BRST symmetry of the GZ action is re-established. In sec. IV, we derive the various Ward identities useful in the proof of renormalizability of the theory. In Sec. V, we conclude the results and make future remarks.

II. THE BLG THEORY IN $\mathcal{N} = 1$ SUPERSPACE: SHORT REVIEW

In this section, first of all we review the construction of BLG theory in $\mathcal{N} = 1$ superspace. To write the action, we first introduce 4-index structure constants f^{ABCD} associated with a trilinear antisymmetric

product [1],

$$[T^A, T^B, T^C] = f_D^{ABC} T^D, \quad (1)$$

where T^A 's are the generators of this Lie 3-algebra. A generalization of the trace, taken over the three-algebra indices, provides an appropriate '3-algebra metric': $h^{AB} = \text{Tr}(T^A T^B)$, which may raise and lower the indices. Totally anti-symmetric in nature, i.e. $f^{ABCD} = f^{[ABCD]}$, these structure constants satisfy the fundamental (Jacobi) identity,

$$f^{[ABC}{}_G f^{D]EG}{}_H = f^{AEF}{}_G f^{BCDG} - f^{BEF}{}_G f^{ACDG} + f^{CEF}{}_G f^{ABDG} - f^{DEF}{}_G f^{ABCG} = 0. \quad (2)$$

Another quantity comprised with 4-index structure constants, $C_{EF}^{AB,CD} = f_{[E}^{AB[C} \delta_{F]}^D]$, are antisymmetric in the pair of indices AB and CD and satisfy [44]

$$C_{EF}^{AB,CD} C_{KL}^{GH,EF} + C_{EF}^{GH,AB} C_{KL}^{CD,EF} + C_{EF}^{CD,GH} C_{KL}^{AB,EF} = 0. \quad (3)$$

In order to construct BLG theory in $\mathcal{N} = 1$ superspace, we first define the non-Abelian Chern-Simons action as

$$S_{CS} = -\frac{k}{4\pi} \int d^3x \nabla^2 [f^{ABCD} \Gamma_{AB}^a \Omega_{aCD}], \quad (4)$$

where k is an integer and

$$\Omega_{ABa} = \omega_{aAB} - \frac{1}{6} C_{AB}^{CD,EF} \Gamma_{CD}^b \Gamma_{abEF} \quad (5)$$

$$\omega_{ABa} = \frac{1}{2} D^b D_a \Gamma_{ABb} - \frac{i}{2} C_{AB}^{CD,EF} \Gamma_{CD}^b D_b \Gamma_{aEF} - \frac{1}{6} C_{AB}^{CD,EF} C_{EF}^{LM,NP} \Gamma_{CD}^b \Gamma_{bLM} \Gamma_{aNP}, \quad (6)$$

$$\Gamma_{ABab} = -\frac{i}{2} \left[D_{(a} \Gamma_{ABb)} - i C_{AB}^{CD,EF} \Gamma_{aCD} \Gamma_{bEF} \right]. \quad (7)$$

with the super-derivative defined by $D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b$.

The matter action is given by

$$S_M = -\frac{1}{4} \int d^3x \nabla^2 \left[\nabla^a X_A^{I\dagger} \nabla_a X_I^A + m^2 X_A^{I\dagger} X_I^A + \mathcal{V} \right], \quad (8)$$

where m refers to the mass, which breaks the conformal invariance, and the covariant derivative is given by

$$\nabla_a X^{AI} = D_a X^{AI} + i \Gamma_a^{AB} X_B^I. \quad (9)$$

The potential term \mathcal{V} is defined by $\mathcal{V} = \frac{8\pi}{k} f_{ABCD} \epsilon^{IJKL} [X_I^A X_K^{B\dagger} X_J^C X_L^{D\dagger}]$.

Now, the classical BLG action in $\mathcal{N} = 1$ superspace is given by [45]

$$S_c = S_{CS} + S_M. \quad (10)$$

The fields of the above action transform under the gauge transformation as follows,

$$\delta X^{IA} = i(\Lambda X^I)^A, \quad \delta X^{IA\dagger} = -i(X^{IA\dagger} \Lambda)^A, \quad \delta \Gamma_a^{AB} = (\nabla_a \Lambda)^{AB}. \quad (11)$$

The BLG action (10) remains invariant under these gauge transformations. This implies that there are redundancy in the gauge degrees of freedom of the BLG action and thus all gauge degrees of freedom are not physical. To get rid of such spurious degrees of freedom, we need to fix a gauge before performing any calculations. Here, we fix the gauge with a suitable covariant gauge condition, $G = D^a \Gamma_a^{AB} = 0$. This

gauge fixing condition can be incorporated in the action at the quantum level by adding the following Landau gauge fixing term to the classical action (10),

$$S_{gf} = \int d^3x \nabla^2 [f^{ABCD} b_{AB} D^a \Gamma_{aCD}]_{|}. \quad (12)$$

The induced Faddeev-Popov ghost term, corresponding to the above gauge fixing term, is given by

$$S_{gh} = \int d^3x \nabla^2 [f^{ABCD} \bar{c}_{AB} D^a \nabla_a c_{CD}]_{|}. \quad (13)$$

Now, the effective action for BLG theory in Landau gauge in $\mathcal{N} = 1$ superspace is given as the sum of the classical action to the gauge fixing term and the ghost term,

$$S_{BLG} = S_c + S_{gf} + S_{gh}. \quad (14)$$

This effective action enjoys the following nilpotent BRST symmetry:

$$\begin{aligned} s \Gamma_a^{AB} &= -[\nabla_a c]^{AB}, & s c_{AB} &= \frac{1}{2} C_{AB}^{CD,EF} c_{CD} c_{EF}, \\ s \bar{c}^{AB} &= b^{AB}, & s b^{AB} &= 0, \\ s X^{IA} &= i c^{AB} X_B^I, & s X^{IA\dagger} &= -i X_B^{I\dagger} c^{AB}. \end{aligned} \quad (15)$$

The sum of gauge fixing and the ghost terms is BRST exact, so, it can be expressed in terms of BRST variation,

$$S_{gf} + S_{gh} = s \int d^3x \nabla^2 [f^{ABCD} \bar{c}_{AB} D^a \Gamma_{aCD}]_{|}. \quad (16)$$

In fact, due to the nilpotency of the BRST transformations, the invariance of the effective action S_{BLG} under BRST symmetry is evident.

III. THE BLG THEORY IN GRIBOV-ZWANZIGER FRAMEWORK

In this section, we discuss the BLG theory in the GZ framework. The motivation for such study is to handle covariant gauge fixing correctly as they are not ideal in non-perturbative (IR) regime. Since two equivalent superfields, satisfying the the Landau gauge, connected by a gauge transformation (11), yield

$$\nabla^2 D^a \nabla_a \Lambda^{AB} = 0. \quad (17)$$

Therefore, the existence of infinitesimal copies, even after Faddeev-Popov quantization, is related to the presence of the zero modes of the operator $\nabla^2 D^a \nabla_a \Lambda^{AB}$. To see the zero mode problem, we take the eigenvalues equation

$$\nabla^2 D^a \nabla_a \Lambda^{AB} = \lambda \Lambda^{AB}. \quad (18)$$

For configurations very close to the vacuum (i.e $\Gamma_a^{AB} = 0$), this reduces to

$$(D^2)^2 \Lambda^{AB} = -\partial^2 \Lambda^{AB} = \lambda \Lambda^{AB}, \quad (19)$$

and, therefore, the operator has only positive eigenvalues. However, this can not be guaranteed always and may be displayed negative eigenvalues at larger amplitudes than the vacuum, i.e., sufficiently large Γ_a^{AB} . Hence, we analytically implement the restriction to the Gribov region Ω , defined as the set of field configurations fulfilling the Landau gauge condition, for which the Faddeev-Popov operator $(-f^{ABCD} \nabla^2 D^a \nabla_a (\Gamma_a^{AB}))$ is strictly positive, as

$$\Omega := \{ \Gamma_a^{AB} | D^a \Gamma_a^{AB} = 0, -f^{ABCD} \nabla^2 D^a \nabla_a (\Gamma_a^{AB}) > 0 \}. \quad (20)$$

The restriction to the domain of integration, in the path integral, can be achieved by following the GZ approach, where the inverse of this operator (horizon function) is included in the functional integral in order to compensate the problem. This is achieved by adding the following horizon term to the effective BLG action:

$$S_h = \int d^3x h(x) = \gamma^4 \int d^3x d^3y \nabla^2 \left[C_{AB}^{CD,EF} \Gamma_{aCD}(x) (-f^{ABGH} \nabla^2 D^a \nabla_a)^{-1} C_{GH,EF}^{LM} \Gamma_{LM}^a(y) \right]_l, \quad (21)$$

and the resulting action is being called as the GZ BLG action. Here, the parameter, γ , has the dimension of the mass and is known as the Gribov parameter. This is not a free parameter. This is a dynamical quantity, being determined in a self-consistent way through a gap equation, called the horizon condition,

$$\langle h \rangle = 3\gamma^4 f(N), \quad (22)$$

where $f(N)$ is a some constant number.

The partition function for GZ BLG action is defined by

$$Z_{BLG}^{GZ} = \int_{\Omega} \mathcal{D}\Gamma \mathcal{D}X \mathcal{D}X^\dagger \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}b e^{-S_{BLG}} = \int \mathcal{D}\Gamma \mathcal{D}X \mathcal{D}X^\dagger \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}b e^{-(S_{BLG} + S_h - 3\gamma^4 f(N))}. \quad (23)$$

By localizing the non-local term it can further be rewritten as

$$Z_{BLG}^{GZ} = \int_{\Omega} \mathcal{D}\Gamma \mathcal{D}X \mathcal{D}X^\dagger \mathcal{D}c \mathcal{D}\bar{c} \mathcal{D}b \mathcal{D}\bar{\varphi} \mathcal{D}\varphi \mathcal{D}\bar{\omega} \mathcal{D}\omega e^{-S_{BLG}^{GZ}}, \quad (24)$$

where the GZ BLG action is given by

$$S_{BLG}^{GZ} = S_{BLG} + S_0 + S_\gamma, \quad (25)$$

with the (localized) horizon action

$$S_0 = \int d^3x \nabla^2 \left[\varphi_{aEF}^{AB} (f^{ABCD} \nabla^2 D^a \nabla_a) \varphi^{aCDEF} - \omega_{aEF}^{AB} (f^{ABCD} \nabla^2 D^a \nabla_a) \omega^{aCDEF} - C_{AB,CD}^{GH} (D_a \bar{\omega}_b^{ABLM}) (f^{CDEF} \nabla^a c_{EF}) \varphi_{GHLM}^b \right]_l \quad (26)$$

and

$$S_\gamma = -\gamma^2 \int d^3x \nabla^2 \left[C_{CD,EF}^{AB} \Gamma_{aAB} (\varphi^{aCDEF} + \bar{\varphi}^{aCDEF}) \right]_l - 3\gamma^4 f(N). \quad (27)$$

In this local formulation, the horizon condition (22) takes the following form:

$$\frac{\partial \mathcal{E}}{\partial \gamma^2} = 0, \quad (28)$$

where the vacuum energy \mathcal{E} is defined by $e^{-\mathcal{E}} = Z_{BLG}^{GZ}$. Here, in the absence of γ -dependent term, the action (25) enjoys the following BRST symmetry:

$$\begin{aligned} s \Gamma_a^{AB} &= -[\nabla_a c]^{AB}, & s c_{AB} &= \frac{1}{2} C_{AB}^{CD,EF} c_{CD} c_{EF}, \\ s \bar{c}^{AB} &= b^{AB}, & s b^{AB} &= 0, \\ s X^{IA} &= i c^{AB} X_B^I, & s X^{IA\dagger} &= -i X_B^{I\dagger} c^{AB}, \\ s \bar{\omega}_{aEF}^{AB} &= \bar{\varphi}_{aEF}^{AB}, & s \varphi_{aEF}^{AB} &= \omega_{aEF}^{AB}. \end{aligned} \quad (29)$$

The GZ BLG action is, however, not invariant under the above set of BRST transformations, due to the term S_γ , as

$$s S_{BLG}^{GZ} = s S_\gamma = \gamma^2 \int d^3x \nabla^2 \left[C_{AB,CD}^{EF} f^{ABLM} \nabla_a c_{LM} (\varphi_{EF}^{aCD} + \bar{\varphi}_{EF}^{aCD}) - C_{AB}^{CD,EF} \Gamma_a^{AB} \omega_{CDEF}^a \right]_l. \quad (30)$$

Utilizing the BRST variation, we rewrite the GZ BLG action by

$$S_{BLG}^{GZ} = S_c + s \int d^3x \nabla^2 [f^{ABCD} \bar{c}_{AB} D^a \Gamma_{aCD} + \bar{\omega}^{ABEF} (f^{ABCD} D^a \nabla_a) \varphi_{CDEF}^b] + S_\gamma, \quad (31)$$

from which relation (30) becomes apparent.

We construct a BRST invariant action, corresponding to the BRST breaking term S_γ , as follows

$$\begin{aligned} \Sigma_\gamma = & s \int d^3x \nabla^2 [-U_{ab}^{ABCD} f_{ABEF} \nabla^a \varphi_{CD}^{bEF} - V_{ab}^{ABCD} f_{ABCD} \nabla_a \bar{\omega}_{CD}^{bEF} - U_{ab}^{ABCD} V_{ABCD}^{ab} \\ & + T_{ab}^{ABCD} C_{AB,LM}^{EF} f^{LMNO} \nabla^a C_{NO} \bar{\omega}_{EFCO}^b], \end{aligned} \quad (32)$$

where we have introduced 3 new doublets $(U_{ab}^{ABCD}, M_{ab}^{ABCD})$, $(V_{ab}^{ABCD}, N_{ab}^{ABCD})$ and $(T_{ab}^{ABCD}, R_{ab}^{ABCD})$ with the following BRST transformations:

$$\begin{aligned} s U_{ab}^{ABCD} &= M_{ab}^{ABCD}, & s M_{ab}^{ABCD} &= 0, \\ s V_{ab}^{ABCD} &= N_{ab}^{ABCD}, & s N_{ab}^{ABCD} &= 0, \\ s T_{ab}^{ABCD} &= R_{ab}^{ABCD}, & s R_{ab}^{ABCD} &= 0. \end{aligned} \quad (33)$$

In order to make this extended theory reminiscent with the original one, we, therefore, set (at the end) the sources to have such values that $\Sigma_\gamma|_{phy} = S_\gamma$. We have, thus, restored the broken BRST symmetry, which may be helpful to establish the renormalizability of the GZ BLG theory.

Thus, the final BLG action in GZ framework is given by

$$\Sigma_{BLG} = S_{BLG} + S_0 + \Sigma_\gamma, \quad (34)$$

which remains invariant under the BRST transformations given in (29) and (33). It is apparent that the spontaneous breaking of the BRST symmetry is entirely driven by the Gribov parameter. This implies that the breaking is due to the Gribov horizon, which assures that the analysis is truly non-perturbative.

IV. THE WARD IDENTITIES

Now, we should try to find all the possible Ward identities. In order to write the Slavnov-Taylor identity, we first have to couple all nonlinear BRST transformations to the external sources. We find that $\Gamma_a^{AB}, c_{AB}, X_A^I$ and $X_A^{I\dagger}$ transform nonlinearly under the BRST transformation. Therefore, we add the following term to the action Σ_{BLG} :

$$S_{ext} = \int d^3x \nabla^2 \left[-K_a^{AB} (\nabla^a c)_{AB} + \frac{1}{2} L^{AB} C_{AB,CD}^{EF} c^{CD} c_{EF} + i \bar{Y}_{AI} c^{AB} X_B^I - i X_B^{I\dagger} c^{AB} Y_{AI} \right], \quad (35)$$

where K_a^{AB}, L^{AB}, Y_{AI} and \bar{Y}_{AI} are four new sources, invariant under the BRST symmetry s and with the physicality conditions

$$K_a^{AB}|_{phys} = L^{AB}|_{phys} = Y_{AI}|_{phys} = \bar{Y}_{AI}|_{phys} = 0. \quad (36)$$

The enlarged action is, thus, given by

$$\Sigma'_{BLG} = \Sigma_{BLG} + S_{ext}, \quad (37)$$

which is indeed BRST invariant. This action Σ'_{BLG} now enjoys a larger number of Ward identities mentioned below:

- The Slavnov-Taylor identity is given by

$$\mathcal{S}(\Sigma'_{BLG}) = 0, \quad (38)$$

where

$$\begin{aligned} \mathcal{S}(\Sigma'_{BLG}) = & \int d^3x \left(\frac{\delta \Sigma'_{BLG}}{\delta K_a^{AB}} \frac{\delta \Sigma'_{BLG}}{\delta \Gamma_{AB}^a} + \frac{\delta \Sigma'_{BLG}}{\delta L^{AB}} \frac{\delta \Sigma'_{BLG}}{\delta c_{AB}} + \nabla^2 b^{AB} \frac{\delta \Sigma'_{BLG}}{\delta \bar{c}^{AB}} + \nabla^2 \varphi_{aCD}^{AB} \frac{\delta \Sigma'_{BLG}}{\delta \bar{\omega}_{aCD}^{AB}} \right. \\ & + \nabla^2 \omega_{aCD}^{AB} \frac{\delta \Sigma'_{BLG}}{\delta \bar{\varphi}_{aCD}^{AB}} + \nabla^2 M_{ab}^{ABCD} \frac{\delta \Sigma'_{BLG}}{\delta \bar{U}_{ab}^{ABCD}} + \nabla^2 N_{ab}^{ABCD} \frac{\delta \Sigma'_{BLG}}{\delta \bar{V}_{ab}^{ABCD}} \\ & \left. + \nabla^2 R_{ab}^{ABCD} \frac{\delta \Sigma'_{BLG}}{\delta \bar{T}_{ab}^{ABCD}} + \frac{\delta \Sigma'_{BLG}}{\delta Y^{AI}} \frac{\delta \Sigma'_{BLG}}{\delta X_{AI}} - \frac{\delta \Sigma'_{BLG}}{\delta Y^{AI}} \frac{\delta \Sigma'_{BLG}}{\delta X_{AI}^\dagger} \right). \end{aligned} \quad (39)$$

- The $U(f)$ invariance reads

$$\mathcal{U}_{ab}^{CDEF} \Sigma'_{BLG} = 0, \quad (40)$$

with

$$\begin{aligned} \mathcal{U}_{ab}^{CDEF} = & \int d^3x \left[\varphi_a^{ABCD} \frac{\delta}{\delta \varphi_b^{ABEF}} - \bar{\varphi}_b^{ABEF} \frac{\delta}{\delta \bar{\varphi}_a^{ABCD}} + \omega_a^{ABCD} \frac{\delta}{\delta \omega_b^{ABEF}} - \bar{\omega}_b^{ABEF} \frac{\delta}{\delta \bar{\omega}_a^{ABCD}} \right. \\ & - M_{cb}^{ABEF} \frac{\delta}{\delta M_{cCD}^{aAB}} - U_{cb}^{ABEF} \frac{\delta}{\delta U_{cCD}^{aAB}} + N_{ca}^{ABCD} \frac{\delta}{\delta N_{cEF}^{bAB}} + V_{ca}^{ABCD} \frac{\delta}{\delta V_{cEF}^{bAB}} \\ & \left. + R_{cb}^{ABEF} \frac{\delta}{\delta R_{cCD}^{aAB}} + T_{cb}^{ABEF} \frac{\delta}{\delta T_{cCD}^{aAB}} \right]. \end{aligned} \quad (41)$$

- The Landau gauge condition is given by

$$\frac{\delta \Sigma'_{BLG}}{\delta b^{AB}} = \nabla^2 D^a \Gamma_{aAB}. \quad (42)$$

- The antighost equation of motion yields

$$\frac{\delta \Sigma'_{BLG}}{\delta \bar{c}^{AB}} + D^a \frac{\delta \Sigma'_{BLG}}{\delta K_a^{AB}} = 0. \quad (43)$$

- The linearly broken, local equation of motion of φ_{aCD}^{AB}

$$\frac{\delta \Sigma'_{BLG}}{\delta \bar{\varphi}_{aCD}^{AB}} + D_b \frac{\delta \Sigma'_{BLG}}{\delta M_{baCD}^{Ab}} + C_{AB}^{EF,LM} T_{bEF}^{aCD} \frac{\delta \Sigma'_{BLG}}{\delta K_b^{LM}} = \nabla^2 C_{AB,GH}^{EF} \Gamma_b^{GH} V_{EF}^{baCD}. \quad (44)$$

The local, linearly broken, equation of motion of ω_{aCD}^{AB}

$$\frac{\delta \Sigma'_{BLG}}{\delta \bar{\omega}_{aCD}^{AB}} + D_b \frac{\delta \Sigma'_{BLG}}{\delta N_{baCD}^{Ab}} + C_{AB}^{EF,LM} \bar{\omega}_{EF}^{aCD} \frac{\delta \Sigma'_{BLG}}{\delta \bar{b}^{LM}} = \nabla^2 C_{AB,GH}^{EF} \Gamma_b^{GH} U_{EF}^{baCD}. \quad (45)$$

- The exact \mathcal{R}_{aCD}^{bEF} symmetry reads

$$\begin{aligned} \mathcal{R}_{aCD}^{bEF} = & \int d^3x \left(\varphi_{aCD}^{AB} \frac{\delta}{\delta \omega_{bEF}^{AB}} - \bar{\omega}_{AB}^{bEF} \frac{\delta}{\delta \bar{\varphi}_{AB}^{aCD}} + V_{caCD}^{AB} \frac{\delta}{\delta N_{cbEF}^{AB}} - U_{cAB}^{bEF} \frac{\delta}{\delta M_{cAB}^{aCD}} \right. \\ & \left. + T_{caCD}^{AB} \frac{\delta}{\delta R_{cbEF}^{AB}} \right). \end{aligned} \quad (46)$$

If we turn to the quantum level, these symmetries can be used to characterize the most general (allowed) BRST invariant counter terms.

V. CONCLUSION

The M2-branes worldvolume theory have the following continuous symmetries: 16 supersymmetries, $SO(8)$ R-symmetry, nontrivial gauge symmetry and conformal symmetry. The multiple M2-branes described by the BLG theory, which is based on Lie 3-algebras imposing totally antisymmetric triple product (or 3-commutator). Though the BLG theory possess conformal invariance, the mass deformed BLG theory is no more a conformal invariant theory and can get the dynamics through non-vanishing β -function. So, it is important to investigate the mass deformed BLG theory in GZ framework to investigate the theory in non-perturbative regime. Based on such reasoning, this is possible to get a dynamical meaning to the Gribov parameter.

We have considered the BLG theory with mass term in $\mathcal{N} = 1$ superspace quantized in Landau gauge in GZ framework. To avoid the Gribov copies from the LG theory in IR regime, a suitable non-local horizon term restricting theory to the first Gribov horizon, has been added to the effective action. This non-local horizon term has further been localized by introducing a suitable quartet of auxiliary fields. Further, the BRST symmetry of the BLG theory in GZ framework has been addressed, where the γ -dependent breaks the BRST invariance. The BRST broken γ -dependent terms are extended further with three pairs of sources to restore the BRST invariance. The various Ward identities for such model, which help to make the theory renormalizable, have been demonstrated. With the help of these sets of Ward identities, one can compute easily the suitable counter terms to absorb the divergences. We believe that the present observation will improve our current understanding of the issue of Gribov problem in the supersymmetric Chern-Simons theory with Lie 3-algebras. It would also be interesting to convert the soft BRST breaking of GZ BLG model in to the linear breaking, which guarantees the renormalizability of the theory. Because the Quantum Action Principle suggests that the linearly broken BRST symmetry can be directly converted into a suitable set of useful Slavnov-Taylor identities.

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